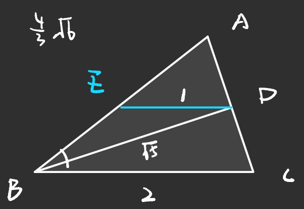
△ABC. $c=\frac{4}{3}\sqrt{6}$. $\cos B=\frac{\sqrt{6}}{6}$ AC中线 $BD=\sqrt{5}$. 求 $\sin A$.



△BDE中 由余弦定理

$$\cos(\pi-B) = -\frac{\sqrt{6}}{6} = \frac{\frac{8}{3}+DE^2-5}{2\cdot\frac{2\sqrt{6}}{3}\cdot DE}$$

$\Rightarrow 3DE^2+4DE-7=0.$

$\Rightarrow DE=1$ (舍负)

在△ABC中. 由余弦定理

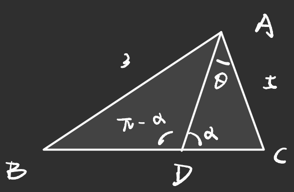
$$AC^2 = \frac{32}{3}+4-2\cdot\frac{4}{3}\sqrt{6}\cdot 2\cdot\frac{\sqrt{6}}{6} = \frac{28}{3}$$

$$AC = \frac{2\sqrt{7}}{\sqrt{3}}$$

在△ABC中. 由正弦定理 $\frac{AC}{\sin B} = \frac{BC}{\sin A} \Rightarrow \sin A = \frac{\sqrt{14}}{14}$

角平分线

AD为角平分线 AB=3. AC=5. ∠BAC=120° 求AD



等面积法 $S_1+S_2=S.$

$$\frac{1}{2}3\cdot AD\sin\angle BAD+\frac{1}{2}5\cdot AD\cdot\sin\angle CAD = \frac{1}{2}\times 3\times 5\sin\angle BAC.$$

角平分线定理 $\frac{AB}{BD}=\frac{AC}{CD}$

证明

$$\frac{S_1}{S_2}=\frac{BD}{DC}=\frac{\frac{1}{2}BD\cdot AD\sin\theta}{\frac{1}{2}AC\cdot AD\cdot\sin\theta}$$

△ADC中 $\frac{AC}{\sin\alpha}=\frac{DC}{\sin\theta}$

△ABD中 $\frac{AB}{\sin(\pi-\alpha)}=\frac{BD}{\sin\theta}$ $\Rightarrow \frac{AC}{AB}=\frac{DC}{BD}$

$BC^2 = 9+25-2\cdot3\cdot5\cos120^\circ = 49.$